
प्रबलित कंक्रीट की खोल संरचना और मुड़ी
हुए प्लेट के डिज़ाइन के लिए मानदंड
(दूसरा पुनरीक्षण)

**Criteria for Design of Reinforced
Concrete Shell Structures and
Folded Plates**
(Second Revision)

ICS 91.080.40

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FOREWORD

This Indian Standard (Second Revision) was adopted by the Bureau of Indian Standards after the draft finalized by the Special Structures Sectional Committee had been approved by the Civil Engineering Division Council.

Shells and folded plates belong to the class of stressed-skin structures which, because of their geometry and small flexural rigidity of the skin, tend to carry loads primarily by direct stresses acting in their plane. Wherever shell is referred to in this standard, it refers to thin shell. On account of multiplicity of the types of reinforced concrete shell and folded plate structures used in present day building practice for a variety of applications demanding roofing of large column-free area, it is not practicable to lay down a rigid code of practice to cover all situations. Therefore, this standard lays down certain general recommendations for the guidance of designers.

Cylindrical shells have been in use in building construction for the past many decades. Well developed theories exist for their analysis. Labour-saving short-cuts, such as, charts and tables for their design, are also available.

Although shells of double curvature, with the exception of domes, have been introduced on a large scale comparatively recently into building construction, these are likely to be used more and more in future. Being non-developable surfaces, they are more resistant to buckling than cylindrical shells and in general, require less thickness. This saving in materials is, however, often offset by the relatively expensive shuttering required for casting them.

Among the doubly-curved shells, the hyperbolic paraboloid and the conoid have, however, the advantage of less expensive shuttering because their ruled surfaces can be formed by straight plank shuttering.

Folded plates are often competitive with shells for covering large column-free areas. They usually consume relatively more materials compared to shell but this disadvantage is often offset by the simpler framework required for their construction.

This standard was first published in 1962 and subsequently revised in 1988. The present revision is based on the developments in the design of shell and folded plate structures subsequently, and to include more rigorous methods of analysis, and numerical methods, such as, finite element methods, which have become available to enforce more rational criteria of design. Some of the significant changes incorporated in this revision are as follows:

- a) New terms and their definitions have been incorporated;
- b) Some of the existing definitions have been modified;
- c) New informative and non-mandatory Annex E, for incorporating analysis using numerical methods has been included;
- d) Guide to finite element modeling of reinforced concrete structures has been included in Annex F;
- e) New informative but non-mandatory Annex G, for response transformation from numerical methods has been included;
- f) Acceptability of approximate methods of analysis has been added;
- g) Shells with prestressed concrete have been addressed; and
- h) Various provisions on materials have been modified in line with IS 456 : 2000 'Plain and reinforced concrete — Code of practice (*fourth revision*)'.

(Continued on third cover)

Indian Standard

CRITERIA FOR DESIGN OF REINFORCED CONCRETE SHELL STRUCTURES AND FOLDED PLATES

(Second Revision)

1 SCOPE

1.1 This standard lays down recommendations for the classification, dimensional proportioning, analysis and design of cast *in-situ*, reinforced concrete thin shells and folded plates.

1.2 All provisions of IS 456 not specifically excluded, and not in conflict with provisions of this standard, shall apply to thin shell concrete structures. Provisions of this standard shall govern for thin shell structures, including ribs and edge members.

1.3 Wherever prestressed concrete is used, in addition to the stipulations of present standard, provisions of IS 1343 shall apply, for pre-tensioned and post-tensioned type of shells, as the case may be.

1.4 This standard does not deal with construction practices relating to these structures which have been separately dealt in IS 2204.

2 REFERENCES

The Indian Standards listed in Annex A contain provisions, which through reference in this text, constitute provisions of this standard. At the time of publication the editions indicated are valid. All standards are subject to revision and parties to agreement based on this standard are encouraged to investigate the possibility of applying the most recent editions of the standards listed in Annex A.

3 TERMINOLOGY

For the purpose of this standard, the following definitions shall apply.

3.1 Anticlastic Shells — A shell is said to be anticlastic shells if its Gauss curvature is negative.

3.2 Asymmetrical Cylindrical Shells — Cylindrical shells which are asymmetrical about the crown.

3.3 Barrel Shells — Cylindrical shells which symmetrical about the crown (see Fig. 1).

3.4 Butterfly Shells — Butterfly shells are those which consist of two parts of a cylindrical shell joined together at their lower edges (see Fig. 2).

3.5 Chord Width — The chord width is the horizontal projection of the arc of the cylindrical shell.

3.6 Continuous Cylindrical Shells — Cylindrical shells which are longitudinally continuous over the traverses (see Fig. 3).

NOTE — Doubly-curved shells continuous in one or both directions may be termed as continuous shells.

3.7 Cylindrical Shells — Shells in which either the directrix or generatrix is a straight line.

3.8 Double Curved — A shell is said to be doubly curved if its Gauss curvature is definite. Such shells are non-developable.

3.9 Edge Member — A member provided at the edge of a shell.

NOTE — Edge members increase the rigidity of the shell edge and help in accommodating the reinforcement.

3.10 Elastic Analysis — An analysis of deformations and internal forces based on equilibrium, compatibility of strains, and assumed elastic behavior, and representing, to a suitable approximation, the three-dimensional action of the shell together with its auxiliary members. For an elastic analysis, concrete may be assumed as uncracked, homogeneous, and isotropic. Elastic behavior is the commonly accepted basis for determining stresses, displacements, and stability of thin shells

3.11 End Frames or Traverses — End frames or traverses are structures provided to support and preserve the geometry of the shell.

NOTE — They may be solid diaphragms, arch ribs, portal frames or bowstring girders. Where a clear soffit is required, especially for the use of movable formwork, the end frames may consist of upstand ribs.

3.12 Folded Plates — Folded plates consist of a series of thin plates, usually rectangular, joined

monolithically along their common edges and supported on diaphragms. They are also known as hipped plates.

Folded plates may be prismatic, non-prismatic or faceted. The first two types consist generally of planar thin slabs joined along their longitudinal edges to form a beam-like structure spanning between supports. Faceted folded plates are made up of triangular or polygonal planar thin slabs joined along their edges to form three-dimensional spatial structures.

NOTE — Shapes of folded plates and their applications

A few of the commonly used shapes of folded plates are shown in Fig. 4. The simplest is V-shaped unit (Fig. 4A) but this may not provide enough area of concrete at the top and bottom to resist the compressive forces due to bending and to accommodate the reinforcement. The trough-shaped or the trapezoidal unit (Fig. 4B and Fig. 4E) eliminates these disadvantages. Asymmetrical sections of the 'Z' shape (Fig. 4C) provided with window glazing between two adjacent units serve as north-light roofs for factory buildings. The shape shown in Fig. 4D is obtained by replacing the curved cross-section of a cylindrical shell by a series of straight plates. This has the advantage of greater structural depth compared to other shapes. Butterfly type of folded plates shown in Fig. 4F are also employed to cover factory roofs as there are provisions for window glazing. Tapering plates are also used as roofs mainly for aesthetic reasons. Hipped plate structures of the pyramidal types are used for tent-shaped roofs, cooling towers, etc.

3.13 Gauss Curvature — The product of the two principal curvatures, $1/R_1$ and $1/R_2$ at any point on the surface of the shell.

3.14 Generatrix, Directrix — A curve which moves parallel to itself over a stationary curve generates a surface. The moving curve is called the generatrix and the stationary curve the directrix. One of them may be a straight line.

NOTE — The common curves used for cylindrical shells are, arc of a circle, semi-ellipse, parabola, catenary and cycloid.

3.15 Inelastic Analysis — An analysis of deformations and internal forces based on equilibrium, nonlinear stress-strain relations for concrete and reinforcement, consideration of cracking and time-dependent effects, and compatibility of strains. The analysis shall represent to a suitable approximation three-dimensional action of the shell together with its auxiliary members.

3.16 Junction Member — The common edge member at the junction of two adjacent shells.

3.17 Multiple Cylindrical Shells — A series of parallel cylindrical shells which are transversely continuous.

3.18 North-Light Shells — Cylindrical shells with two springing at different levels and having provisions for north-light glazing (see Fig. 5).

3.19 Radius — Radius at any point of the shell in one of the two principal directions.

NOTE — If cylindrical shell of a circular arc is used, the radius of the arc is the radius of the shell. In other cases, the radius R at any point is related to the radius R_0 at the crown by $R = R_0 \cos(\theta)$, where θ is the angle of inclination of the tangent to the curve at that point. The parabola, value of n is 1, - 2 and - 3 for cycloid, catenary and parabola, respectively. For an ellipse:

$$R = \frac{a^2 b^2}{(a^2 \sin^2 \theta + b^2 \cos^2 \theta)^{3/2}}$$

where, a and b are the semi-major and semi-minor axes, respectively, and is the slope of the tangent at the point.

3.20 Ribbed Shells — Spatial structures with material placed primarily along certain preferred rib lines, with the area between the ribs filled with thin slabs or left open. Ribbed shells generally have been used for larger spans where the increased thickness of the curved slab alone becomes excessive or uneconomical. Ribbed shells are also used because of the construction techniques employed and to enhance the aesthetic impact of the completed structure.

3.21 Rise — The vertical distance between the apex of the curve representing the centre line of the shell and the lower most springing.

3.22 Ruled Surfaces — Surfaces which can be generated entirely by straight lines. The surface is said to be 'singly ruled' if at every point, a single straight line only can be ruled and 'doubly ruled' if at every point, two straight lines can be ruled. Cylindrical shells, conical shells and conoids are examples of singly ruled surfaces; hyperbolic paraboloids and hyperboloids of revolution of one sheet are examples of doubly ruled surface (see Fig. 6).

3.23 Semi Central Angle — Half the angle subtended by the arc of a symmetrical circular shell at the centre.

3.24 Shells — Three-dimensional spatial structures made up of one or more curved slabs or folded plates whose thicknesses are small compared to their other dimensions. Thin shells are characterized by their three-dimensional load-carrying behavior, which is determined by the geometry of their forms, by the manner in which they are supported, and by the nature of the applied load. Thin shells are those in which the radius to thickness ratio should not be more than 20.

3.25 Shells of Revolution — Shells which are obtained when a plane curve is rotated about the axis of symmetry. Examples are segmental domes, cones, paraboloids of revolution, hyperboloids of revolution, etc (see Fig. 7).

3.26 Shells of Translation — Shells which are obtained when the plane of the generatrix and the directrix are at right angles. Examples are cylindrical shells, elliptic paraboloids, hyperbolic paraboloids, etc (see Fig. 8).

3.27 Singly Curved — A shell is said to be singly curved if its one principal curvature is zero (hence, Gauss curvature is zero) It is also known as developable shell, since, such surfaces can be developed by bending a plane surface.

3.28 Span — The span of a cylindrical shell is the distance between the centre lines of two adjacent end frames of traverses (see Fig. 1).

3.29 Synclastic Shell — A shell is said to be synclastic shell if its Gauss curvature is positive.

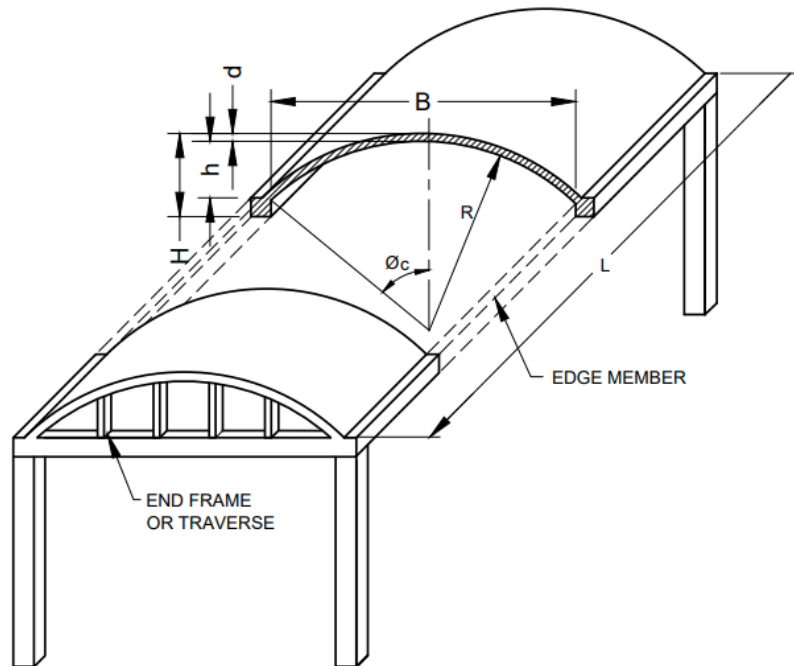


FIG. 1 SINGLE BARREL SHELL

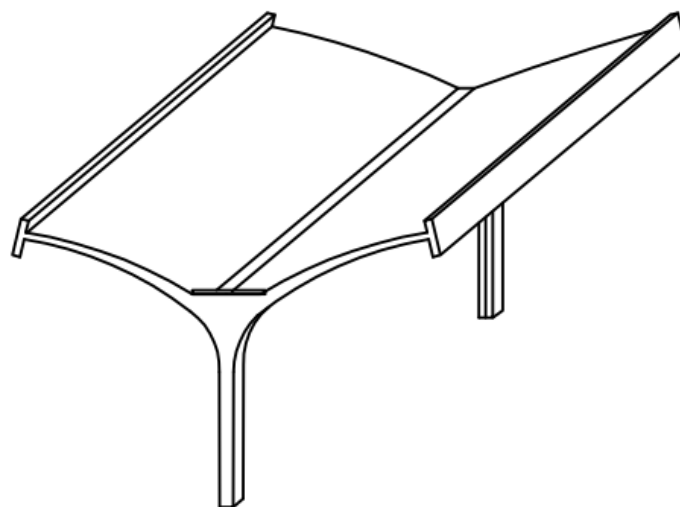


FIG. 2 BUTTERFLY SHELL

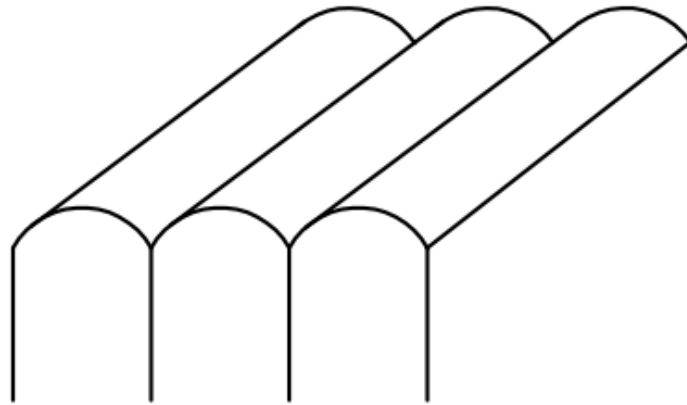
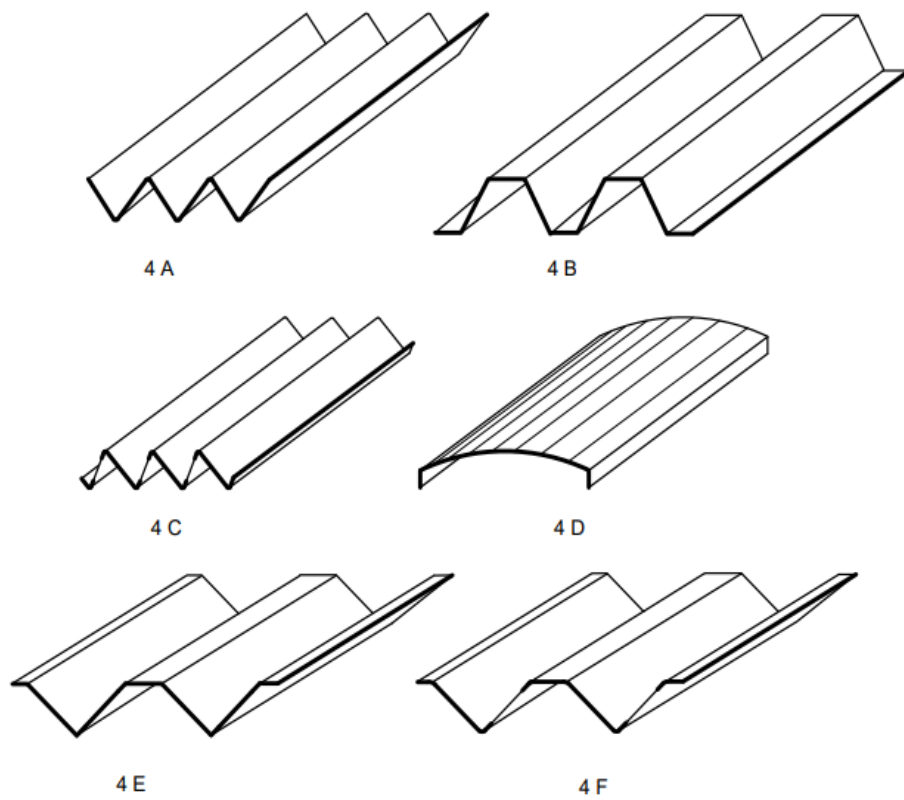


FIG. 3 MULTIPLE BARREL SHELL



NOTE — Diaphragms are not shown.

FIG. 4 FOLDED PLATES

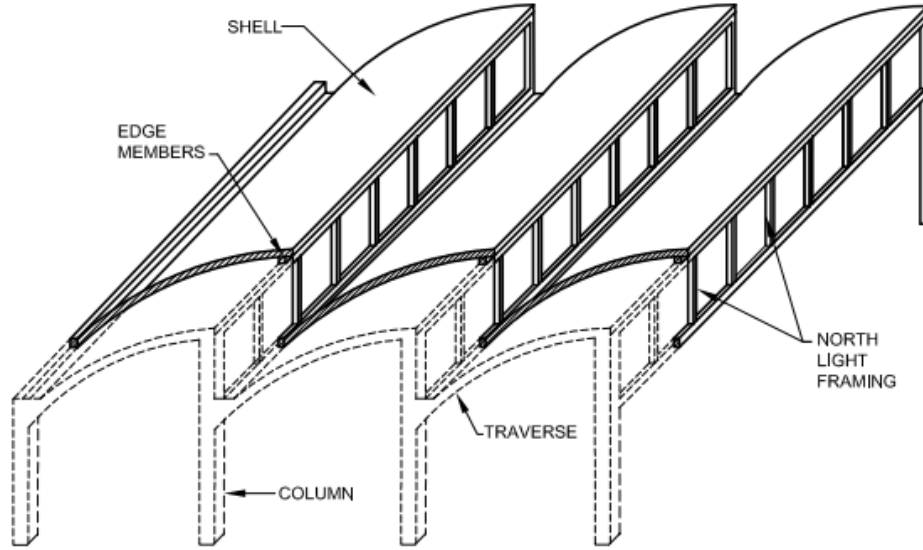


FIG. 5 NORTH — LIGHT CYLINDRICAL SHELLS

4 NOTATIONS

4.1 For the purpose of this standard, unless otherwise defined in the text, the following notations shall have the meaning indicated against each:

a	= Semi-major axis of an elliptical shell;
B	= Chord width;
b	= Semi-minor axis of an elliptical shell;
D	= Flexural rigidity;
d	= Thickness of shell;
E_c	= Modulus of elasticity of concrete (long term);
E_s	= Modulus of elasticity of steel;
F	= Stress function which gives the in-plane stress in doubly-curved shells when bending is also considered;
f_{ck}	= Characteristic strength of concrete;
f_{cr}	= Critical buckling stress;
h	= Rise of shell;
H	= Total depth of shell, measured from the crown of the shell to the bottom of the edge member;
L	= Span;
m_x	= Bending moment in the shell in the x-direction;
m_y	= Bending moment in the shell in the y-direction;

m_{xy}	= Twisting moment in the shell;
n_x, n_y and n_{xy}	= Real membrane stresses in the shell;
n_{xp}, n_{yp} and n_{xyp}	= Projected membrane forces;
P	= Permissible buckling load per unit area of the surface of doubly-curved shells;
p	= $\frac{\partial z}{\partial x}$;
q	= $\frac{\partial z}{\partial y}$;
r	= $\frac{\partial^2 z}{\partial x^2}$;
s	= $\frac{\partial^2 z}{\partial x \cdot \partial y}$;
t	= $\frac{\partial^2 z}{\partial y^2}$;
R	= Radius;
R_1 and R_2	= Principal radii of curvature at any point on the surface of shell;
S	= Shear stress;
T_x	= Normal stress in the x-direction;
T_y	= Normal stress in the y-direction;
W_x, W_y and W_z	= Real forces on unit area of the shell in the x, y and z direction;
W	= Deflection in the direction of z-axis;

X, Y and Z	=	Fictitious forces on unit projected area of the shell in the x, y and z -directions;
x, y and z	=	Axes of co-ordinates;
ϕ	=	Stress function used in the membrane analysis of doubly-curved shell;
θ	=	Angle of inclination of tangent to the curve at any point;
ϕ_c	=	Semi-central angle of a symmetrical circular cylindrical shell;
ρ and κ	=	Aas Jakobsen's parameters for cylindrical shells;
ν	=	Poisson's ratio; and
∇^2	=	$\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right]$

5 CLASSIFICATION OF SHELLS

5.1 General

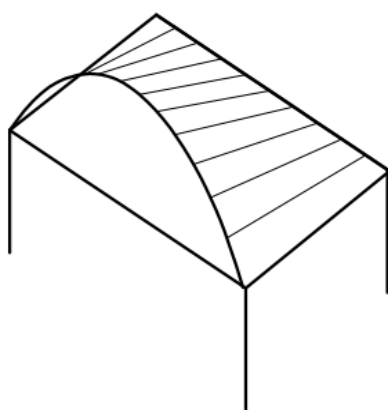
Shells may be broadly classified as 'singly-curved' and 'doubly-curved'. This is based on Gauss

curvature. The gauss curvature of singly-curved shells is zero because one of their principal curvatures is zero. They are, therefore, developable. Doubly-curved shells are non-developable and are classified as synclastic or anticlastic according as their Gauss curvature is positive or negative.

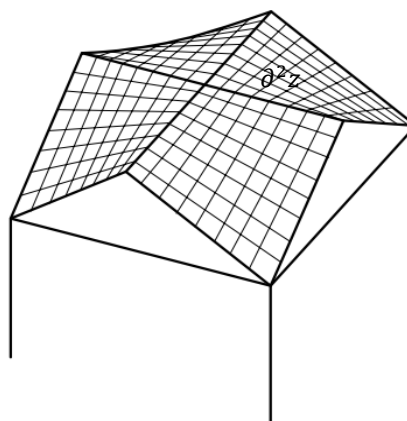
5.1.1 The governing equations of membrane theory of singly curved shell are parabolic. It is elliptic for synclastic shells and hyperbolic for anticlastic shells. If $z = f(x, y)$ is the equation to the surface of a shell, the surface will be synclastic, developable or anticlastic according as, $s^2 - r \cdot t \gtrless 0$, where r, s and t are as defined in 4.1.

5.1.2 There are other special types of doubly curved shells, such as, funicular shells, which are synclastic and anticlastic in parts and corrugated shells which are alternately synclastic and anticlastic. The gauss curvature for such shells is positive where they are synclastic and negative where they are anticlastic.

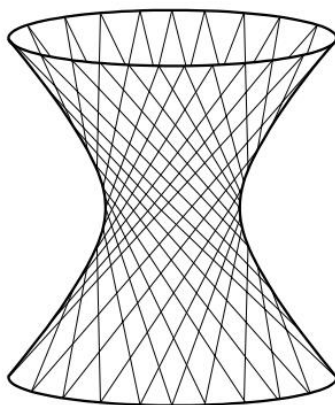
5.2 The detailed classification of shell structures is given in Annex B.



6A CONOID

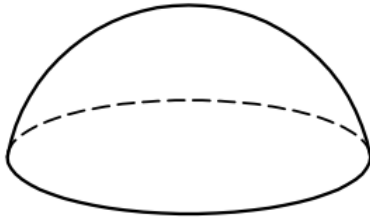


6B HYPERBOLIC PARABOLOID

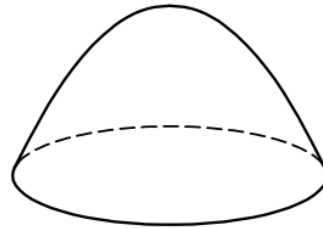


6C HYPERBOLIC OF REVOLUTION OF ONE SHEET

FIG. 6 RULED SURFACES

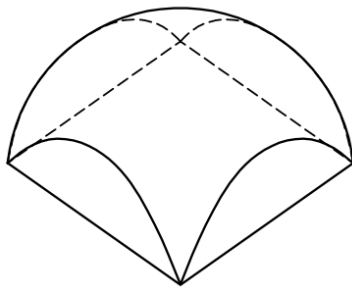


7A SEGMENTAL DOME

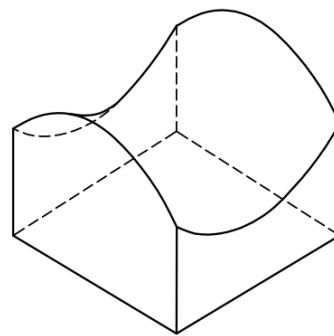


7B PARABOLOID OF REVOLUTION

FIG. 7 SHELL OF REVOLUTION



8A ELLIPTIC PARABOLOID



8B HYPERBOLIC PARABOLOID

FIG. 8 SHELL OF TRANSLATION

6 MATERIALS

6.1 Concrete

Controlled concrete shall be used for all shell and folded plate structures. The concrete is of minimum grade M20. The quality of materials used in concrete, the methods of proportioning and mixing the concrete shall be done in accordance with the relevant provisions of IS 456.

NOTE — High cement content mixes are generally undesirable as they shrink excessively giving rise to cracks.

6.2 Steel

The steel for the reinforcement shall be:

- Mild steel and medium tensile steel bars and hard-drawn steel wire conforming to IS 432 (Part 1) and IS 432 (Part 2);
- Hard-drawn steel, wire fabric for concrete reinforcement conforming to IS 1566; and
- High strength deformed bars conforming to IS 1786.

Specified/design yield strength of non-prestressed reinforcement shall not exceed 415 MPa, to minimize cracking of thin shells.

6.2.1 Welding may be used in reinforcement in accordance with IS 456.

7 LOADS

7.1 Unless otherwise specified, shells and folded plates shall be designed to resist the following load combinations:

- Dead load,
- Dead load + appropriate imposed load or snow load,
- Dead load + appropriate imposed load + wind effects, and
- Dead load + appropriate imposed load + seismic effect.

7.2 Dead loads shall be calculated on the basis of the unit weights taken in accordance with IS 875 (Part 1).

7.3 Imposed loads, wind loads and snow loads shall be taken as specified in IS 875 (Parts 2 to 4).

7.4 Seismic effects shall be taken in accordance with IS 1893 (Part 1).

7.5 Where concentrated loads occur, special considerations should be given in analysis and design.

7.6 Secondary loads due to diurnal and seasonal variation of temperature should be considered in the design load combinations, in line with IS 875 (Part 5).

8 SELECTION OF DIMENSIONS

8.1 Thickness

8.1.1 Thickness of Shells

The thickness of a shell and its reinforcement shall be proportioned for the strength and serviceability requirements of IS 456. Strength design of shell slabs for membrane and bending forces shall be based on the distribution of stresses and strains as determined from either elastic or an inelastic analysis.

Thickness of shells shall not normally be less than 50 mm if singly curved and 40 mm if doubly-curved. This requirement does not, however, apply to small precast concrete shell units in which the thickness may be less than that specified above, but it shall in no case be less than 25 mm (*see* IS 6332).

8.1.1.1 The reinforcement shall have a minimum clear cover of 15 mm or its nominal size whichever is greater.

8.1.2 Shells are usually thickened for some distance from their junction with edge members and traverses. The thickening is usually of the order of 30 percent of the shell thickness. It is, however, important to note that undue thickening is undesirable. In the case of singly-curved shells, the distance over which the thickening at the junction of the shell and traverse is made should be between $0.38\sqrt{R.d}$ and $0.76\sqrt{R.d}$, where R and d are the radius of the shell directrix and the thickness, respectively. The thickening of shell at straight edges shall depend on the transverse bending moment. For doubly-curved shells, this distance will depend upon the geometry of the shell and the boundary conditions as the extent of bending penetration is governed by these factors.

8.1.3 Thickness of Folded Plates

The thickness of folded plates shall not normally be less than 75 mm. However, considering the environment conditions, fire resistance, durability requirements, and number of reinforcement layers to be provided, requirement of cover as per IS 456, minimum thickness may be suitably increased.

8.2 Other Dimensions

8.2.1 Cylindrical Shells

8.2.1.1 The span should preferably be less than 30 m. Shells longer than 30 m will involve special design considerations, such as the application of prestressing techniques.

8.2.1.2 The width of the edge member shall generally be limited to three times the thickness of the shell.

8.2.1.3 The radius of shell structures shall be selected keeping acoustic requirements in view. Coincidence of the centre of curvature with the working level should be avoided unless suitable acoustic correction is made. It is, however, important to note that even where coincidence of centre of curvature with the working level is avoided, acoustic treatment may be necessary in certain cases.

8.2.1.4 A single cylindrical shell whose span is larger than three times the chord width shall have a total depth, H , between 1/12 and 1/6 of its span (the former value being applicable to smaller spans). The rise in the case of a shell without edge members shall not be less than 1/10 of its span.

8.2.1.5 For a shell with chord width larger than three times the span, the rise of the shell shall not be less than 1/8 of its chord width.

8.2.1.6 The chord width of shells shall preferably be restricted to six times the span as otherwise arch action is likely to predominate.

8.2.1.7 The semi-central angle shall preferably be between 30° and 40°.

NOTE — Keeping the semi-central angle between these limits is advisable for the following reasons:

- a) If the angle is below 40°, the effect of wind load on the shell produces only suction; and
- b) With slopes steeper than 40°, backforms may become necessary.

Within these limits the semi-central angle shall be as high as possible consistent with the functional requirements.

8.2.2 Folded Plates

8.2.2.1 For folded plates of type shown in Fig. 4D, the selection of depth may be based on the rules applicable to cylindrical shells. With other shapes, such as, the 'V' or the trough, the depth may be taken as about 1/15 of the span for preliminary designs.

8.2.2.2 The angle of inclination of the plates to horizontal shall be limited to about 40° for *in-situ*

construction in order to facilitate placing of concrete without the use of the back forms.

9 ANALYSIS

9.1 General

Shells may be analyzed either by linear elastic analysis based on theory of elasticity or yield line theory. Methods based on yield line theory for shells may be used along with model tests to check the load carrying capacity.

9.1.1 The finite element method has become a practical method of analysis for all types of structures. Many common and important features of shell and folded plate structures that cannot be considered by classical methods can be analyzed satisfactorily by the finite element method, such as:

- a) Complex support or boundary conditions;
- b) Openings large enough to disturb global stress distribution;
- c) Irregular surface geometry;
- d) Highly variable or localized loads;
- e) Tapering folded plates or boxes or silo and bunker bottoms;
- f) Branching shells;
- g) Large deformations;
- h) Heavy and eccentric stiffeners;
- j) Thermoelastic strains;
- k) Elastoplastic, viscoelastic or any inelastic behaviour;
- m) Material non-homogeneity;
- n) Irregular surface geometry;
- p) Sudden changes in curvature;
- q) Shells under dynamic wind action; and
- r) Possible effect of settlement.

9.1.2 When one or more of these complexities occur in shell structures, it is advisable to use finite element method, at least for a final acceptance of the design. Even normal shell structures of spans larger than 30 m should be analyzed by finite element method, if it is expected that there would be serious and significant structural participation in the shell behaviour by the supporting units, such as, edge and intermediate beams, stiffeners, and or intermediate traverses specially flexible traverses cable supports, columns, etc. Provisions of Annex E and Annex F may be adopted, wherever necessary.

9.1.3 For many shells, finite strip method (FSM) (a particular form of finite element method) of analysis is easier to apply and also more economical to use than the finite element method. All types of prismatic folded plates and cylindrical shells can be handled by FSM since these shells can be discretized into long strip elements. Shells of revolution can also be efficiently analyzed by this method, after discretization of such shells into finite ring elements.

Since FSM can be used even when the loads are not uniform, it may be advisable to use the method even for simple shells that are amenable to analysis by common classical methods.

9.1.4 Approximate methods of analysis shall be permitted, where it can be shown that such methods provide a safe basis for design. Such solutions should include both membrane and bending effects, and satisfy conditions of compatibility and equilibrium. Approximate solutions that satisfy statics, but not the compatibility of strains may be used only when extensive experience has proved that safe designs have resulted from their use. Such methods include beam-type analysis for barrel shells and folded plates having large ratios of span to either width or radius of curvature, simple membrane analysis for shells of revolution, and others, in which the equations of equilibrium are satisfied, while the strain compatibility equations are not.

9.1.5 Inelastic analyses shall be permitted to be used where it can be shown that such methods provide a safe basis for design.

9.1.6 Equilibrium checks of internal resistances and external loads shall be made to ensure consistency of results.

9.1.7 An ultimate strength analysis may be used only as a check on the adequacy of the design margins. It is not to be used as a sole criterion for design, except where it can be proven to be applicable.

9.1.8 Model Analysis

Models may be used as the basis for a design and/or to check the validity of assumptions involved in a mathematical analysis. When models are used, only those portions which significantly affect the items under study need be simulated. Every attempt must be made to ensure that these tests reveal the quantitative behavior of the prototype structure.

The common classical methods of analysis of shells are mentioned in the following clauses for use for the analysis of common types of shells that are without any of the complexities.

9.2 Cylindrical Shells

9.2.1 Analytical Methods

The analytical methods consist of two parts, membrane analysis and edge disturbance analysis.

9.2.1.1 Membrane analysis

In the membrane analysis, the shell is regarded as a perfectly flexible membrane which is infinite in extent and is assumed to carry loads by means of forces in its plane only. This analysis gives the two normal stress resultants n_x and n_y in the longitudinal

and the transverse directions and the shear stress resultant n_{xy} .

9.2.1.2 Edge disturbance analysis

Shells, in practice, are always limited by finite boundaries where the boundary conditions demanded by the membrane theory are not obtained with the result that a pure membrane state would seldom exist. Edge disturbances emanate from the boundaries, altering the membrane state and causing bending stresses in the shell. These are accounted for by carrying out the edge disturbance analysis. Usually edge disturbance analysis is confined to disturbance emanating from straight edge as any disturbance emanating from curved edges is damped quite fast. Even in the case of disturbance from straight edges, the bending stresses would get damped out more rapidly in shells having chord width larger than the span and would seldom travel beyond the crown, with the result that the effect of the further edge may be ignored without appreciable error.

The superposition of the membrane and the edge disturbance stresses gives the final stress pattern in the shell.

9.2.1.3 Tables for the analysis of circular cylindrical shells

Simplifications in the analysis of shells are possible by systematizing the calculations making use of tables compiled for this purpose (see Annex C).

9.2.2 Applicability of the Methods of Analysis

9.2.2.1 Cylindrical shells with (L/R) ratio less than π shall be analyzed using any of the accepted analytical methods (see Annex C).

In such shells, if ρ (rho) exceeds 10 and κ (kappa) exceeds 0.15, the effect at any point on the shell of the disturbances emanating from the farther edge may be ignored, where:

$$\rho = \sqrt[8]{\frac{12\pi^4 R^2}{L^4 d^2}} \quad \text{and} \quad \kappa = \frac{\pi^2 R^2}{L^2 \rho^2}$$

For shells with ρ (rho) less than 7 and κ (kappa) less than 0.12, the effect of the disturbances from both the edges shall be considered. Shells with ρ (rho) values between 7 and 10 and κ (kappa) between 0.12 and 0.15 are relatively infrequent. However, should such cases arise, the effects of both the edges shall be considered.

9.2.2.2 Cylindrical shells with (L/R) greater than or

equal to π may be treated as beams of curved cross section spanning between the traverses and the analysis carried out using an approximate method known as the beam method (see Annex C) which consists of the following two parts:

- The beam calculation which gives the longitudinal stress resultant n_x and the shear stress resultant n_{xy} , and
- The arch calculation which gives the transverse stress resultant n_y and the transverse moment m_y .

9.2.3 Continuous Cylindrical Shells

9.2.3.1 Analytical methods

In the analytical methods, the problem of continuous shells is solved in two stages. In the first, the shell is assumed to be simply supported over one span and all the stress resultants worked out. In the second stage, corrections for continuity are worked out and are superimposed on the values corresponding to the simply supported span. Long cylindrical shells can be analyzed by approximating the cylindrical profile by a folded plate shape and applying well known analysis methods for continuous folded plates (see Annex C).

9.2.3.2 Beam method

Solution of continuous shell with $(L/R) \geq \pi$ is simpler by the beam method. The bending moment factors are obtained by solving the corresponding continuous beam. Thereafter, analysis can be continued as in 9.2.2.2.

9.3 Doubly-Curved Shells

9.3.1 Membrane Analysis

In the membrane analysis, it is assumed that the shells carry loads by in-plane stress resultants and usually only deep doubly-curved shells behave like membranes. The governing equations for membrane analysis of doubly-curved are usually solved by using stress functions (see Annex D).

9.3.2 Shallow Shells

Shells may be considered shallow if the rise to span ratio is less than or equal to 1/5 and p^2 and q^2 may be ignored in all the expressions. The shorter side shall be considered as the span for this purpose for shells of rectangular ground plan. A shell with a circular ground plan may be considered shallow if the rise does not exceed 1/5 of the diameter.

NOTE — Based on the same consideration of ignoring p^2 , q^2 and pq being very small, it is sometimes suggested that the shells may be treated as shallow, if the surface is such that, the values of p and q do not exceed 1/8 at any point on it. However, in normal cases, for practical purposes, higher values of p and q up to half may be considered as shallow provided the span to rise ratio does not exceed 5.

9.3.3 *Boundary Conditions for Doubly-Curved Shells*

In general, in the membrane analysis of synclastic shells, only one boundary condition is admissible on each boundary. For an anticlastic shell, the boundary conditions have to be specified in a special manner as the characteristic lines of such surfaces play a significant role in the membrane theory. The type of boundary conditions that can be specified depend on whether or not the boundaries of the shell are characteristic lines.

Further, a membrane state of stress can be maintained in a shell only if the boundaries are such that the reactions exerted by the boundary members on the shells correspond to stresses in the shell at the boundaries given by the membrane theory. It is seldom possible to provide boundary conditions which would lead to a pure membrane state of stress in the shell. In most practical cases, a resort to bending theory becomes necessary.

9.3.3.1 Only deep doubly-curved shells behave like membranes and it is only for such shells that a membrane analysis is generally adequate for design (see also 9.1). Bending analysis is necessary for all singly-curved shells and shallow doubly-curved shells. The governing equation for bending analysis of shallow shells are given in Annex D. For important and large structures, edge effects cannot be neglected altogether. Such effects should be taken care by detailed analysis, and adequate detailing by providing additional reinforcement as necessitated by the analysis and design.

9.3.4 *Bending Theory of Doubly-Curved Shells*

The governing equations for bending analysis of shallow doubly-curved shells are given in Annex D.

9.3.5 *Bending Theory of Shells of Revolution, Symmetrically Loaded*

In synclastic shells of revolution, such as, domes which are symmetrically loaded, a more or less membrane state of stress exists. The bending stresses are confined to a very narrow strip close to edge members and get rapidly damped out. The bending stresses in domes can be calculated with sufficient accuracy by approximate methods like Geckler's approximation.

9.3.6 *Funicular Shells*

The shapes of these shells are so chosen that, under uniformly distributed vertical loads, in a membrane state of stress, they develop only pure compression unaccompanied by shear stresses. Thus theoretically no reinforcement will be necessary except in the edge members. Small precast funicular shells

without any reinforcement except in edge beams are suitable for roofs and floors of residential, industrial, and institutional buildings (see IS 6332). For roofs of larger size, in situ construction may be resorted to; in such shells, provision of reinforcement is necessary to take care of the effects of shrinkage, temperature and bending.

9.4 **Folded Plates**

The structural action of folded plates may be thought of as consisting of two parts, the 'slab action' and the 'plate action'. By the slab action, the loads are transmitted to the joints by the transverse bending of the slabs. The slabs, because of their large depth and relatively small thickness, offer considerable resistance to bending in their own planes and are flexible out of their planes. The loads are, therefore, carried to the end diaphragms by the longitudinal bending of the slabs in their own planes. This is known as 'plate action'. The analysis of folded slabs is carried out in two stages.

9.4.1 *Transverse Slab Action Analysis*

The transverse section of the slab, of unit length, is analyzed as a continuous beam on rigid supports. The joint loads obtained from this analysis are replaced by their components in the planes of the slabs and these are known as plate loads.

9.4.2 *In-Plane Plate Action Analysis*

Under the action of 'plate loads' obtained above, each slab is assumed to bend independently between the diaphragms, and the longitudinal stresses at the edges are calculated. Continuity demands that the longitudinal stresses at the common edges of the adjacent slabs be equal. The corrected stresses are obtained by introducing edge shear forces.

9.5 **Expansion Joints**

The expansion joint shall conform to provisions laid down in IS 456. In the case of folded plates, it is recommended that the joint may be located in the ridge slab. In the cases of large spans where it is not feasible to provide expansion joints, effects of shrinkage shall be taken care of in the design.

9.6 **Openings in Shells**

Openings in shells shall preferably be avoided in zones of critical stresses. Small openings of size not exceeding five times the thickness in shells may be treated in the same way as in the case of reinforced concrete structures. For larger openings, detailed analysis should be carried out to arrive at stresses due to the openings.

9.7 In prestressed concrete shells, analysis shall also consider behavior under loads induced during

prestressing, at cracking load, and at factored load. Where tendons are draped within a shell, design shall take into account force components on the shell resulting from the tendon profile not lying in one plane.

9.8 If construction results in deviations from the shape greater than the specified tolerances, an analysis of the effect of the deviations shall be made and any required remedial actions shall be taken to ensure safe behavior.

9.9 Radial tensile stress along the thickness direction should be checked. Radial reinforcement should be provided, when radial tensile stresses are greater than permissible direct tensile stress of concrete (with applicable factor of safety). Radial reinforcement can be single legged link type, and connecting outer and inner reinforcement mesh.

10 ELASTIC STABILITY

10.1 Permissible Stresses

10.1.1 Stresses in steel reinforcement, and concrete for shells and folded plates shall be in accordance with the provisions given in IS 456.

NOTE — If limit state of design is to be used for shells, specialist literature may be consulted.

10.2 Causes of Instability

Instability in a cylindrical shell may be caused by:

- Local buckling in zones submitted to compressive stresses;
- Flattening of shells, known as the 'Brazier Effect', which occurs particularly in shells without edge members; and
- Instability caused by the combined effect of bending and torsion in the shell as a whole. This occurs particularly in asymmetrical shells.

10.3 Buckling in Cylindrical Shells

10.3.1 The permissible buckling stress f_{ac} in cylindrical shells shall be calculated as follows:

$$f_{ac} = \frac{0.25 f_{ck}}{1 + (f_{ck}/f_{cr})}$$

where

f_{ck} = Characteristic strength of concrete at 28 days; and

f_{cr} = Critical buckling stress determined in accordance with (a), (b) and (c) below:

- Shells with $\rho < 7$ and $\kappa < 0.12$: In such shells, buckling is caused by excessive longitudinal compression near the crown of the shell and the critical buckling stress f_{cr} shall be calculated as follows:

$$f_{cr} = 0.2(E_c \cdot d/R)$$

- Shells with $\rho > 10$ and $\kappa > 0.15$: In such shells, the transverse stresses tend to be critical from the point of view of buckling and the critical buckling stress f_{cr} shall be determined as follows:

- For shells with $L < 2.3\sqrt{dR}$:

$$f_{cr} = E_c [3.4(d/L)^2 + 0.025(L/R)]$$

- For shells with $L > 2.3\sqrt{dR}$:

$$f_{cr} = E_c \left[\frac{0.89 \frac{d}{L} \sqrt{\frac{d}{R}}}{1 - 1.18 \sqrt{\frac{d \cdot R}{L}}} \right]$$

where

E_c = $5\,000\sqrt{f_{ck}}$ (as per IS 456) duly modified for long term modulus including the effect of creep;

d = Thickness of the shell; and

R = Radius of curvature.

- Shells with ρ values between 7 and 10 and κ between 0.12 and 0.15 are relatively infrequent. For such shells, formulae given in (a) or (b) shall apply depending upon whether longitudinal or transverse stresses are critical from considerations of elastic stability.

The value of modulus of elasticity of concrete to be used in the above formulae for calculating the buckling stresses should be the value for long term modulus including the effect of creep also.

10.4 Buckling in Doubly-Curved Shells

For spherical shells, the permissible buckling load per unit area of surface, P_{perm} from considerations of elastic stability, is given by:

$$P_{perm} = 0.1E_c \cdot d^2/R^2$$

where

E_c , d and R are as defined in 4.1.

For other types of doubly-curved shells, the permissible buckling load per unit area of surface, P_{perm} shall be calculated from the formula:

$$P_{perm} = 0.1E_c d^2 / (R_1 \cdot R_2)$$

where

R_1 and R_2 are principal radii of curvature at any point, and E_c and d are as defined in 4.1.

10.5 Buckling in Folded Plates

The folded plate may be replaced by the corresponding cylindrical shell where possible and the appropriate formula used to check for elastic stability.

11 DESIGN OF TRAVERSES

11.1 Types of Traverses

Traverses may be solid diaphragms, arches, portal frames, trusses or bowstring girders. For shells with large chord widths, it is advantageous to have trusses in the form of arches, trusses or bowstring girder.

11.1.1 Traverses may be placed below or above the shells. Where a clear soffit is required, especially to facilitate the use of movable formwork, they may be in the form of upstand ribs.

11.1.2 The simplest diaphragms for folded plates are rectangular beams with depth equal to the height of the plate. The diaphragms are subject to the action of the plate loads on one-half of the span of the folded plate.

11.2 Load on Traverses

Traverses shall be designed to carry, in addition to their own weight, reactions transferred from the shell in the form of shear forces, and the loads directly acting on them. For preliminary trial designs, however, the total load on half the span of the structure may be considered as a uniformly distributed vertical load on the diaphragm.

11.3 Design

The shear forces transferred on to the end frames from the shell shall be resolved into vertical and horizontal components and the analysis made by the usual methods. Owing to the monolithic connection between the traverse and the shell, the latter participates in the bending action of the traverse. The effective width of a cylindrical shell that acts along with the traverse may be assumed as $0.38\sqrt{R \cdot d}$ to $0.76\sqrt{R \cdot d}$ on either side in the case of intermediate traverses and on one side in the case of end ones; the higher value being applicable to an infinitely rigid rib that can prevent the shell from rotating and the lower value to a flexible rib. Where

solid diaphragm traverses are used, adequate reinforcement to distribute shrinkage cracking shall be provided throughout the area of the traverses.

11.3.1 In the design of tied arches, it may be necessary to determine the elastic extension of the tie member due to tension and the consequent effect on the horizontal thrust on the arch.

11.3.2 The bottom member of a bowstring girder, or the tie in the case of a tied arch, is usually subjected to heavy tension. Welding or the provision of threaded sleeve couplings (see of IS 456) or laps may be used for joints in the reinforcement rods. Where lapping is done, the length of the overlap shall be as specified in the relevant clause of IS 456 and the composite tension shall be restricted to $0.1 f_{ck}$, where f_{ck} is the characteristics cube strength of concrete at 28 days. Where the composite tension exceeds $0.1 f_{ck}$, the entire length of the lap shall be bound by a helical binder of 6 mm diameter at a pitch not exceeding 75 mm. The joints in the bars shall always be staggered. Prestressing the tension member offers a simple and satisfactory solution. The detailing of inclined or vertical members of trusses or bowstring girders and suspenders of tied arches should be done with great care. The reinforcements in the tie of tied-arches shall be securely anchored at their ends.

11.4 The traverses may be hinged to the columns, except where the traverses and columns are designed as one unit, such as in a portal frame. Provision shall be made in the design of columns to allow for the expansion or contraction of traverses due to temperature changes.

12 DESIGN OF EDGE BEAMS

12.1 Edge beams stiffen the shell edges and act together with the shell in carrying the load of the supporting system. They can either be vertical or horizontal. Vertical beams are usually employed in long cylindrical shells wherein the cylindrical action is predominant. Horizontal beams are employed in short cylindrical shells where transverse arch action is predominant. It is preferable to completely isolate the structural system of the shell structure without adding any other structure to it.

In most of the shell forms, edge beams form part of the shell structure itself. An analysis of the shell structure is carried out along with the edge beam. Analysis and design of edge beams should ensure compatibility of boundary conditions at the shell edge. Analysis should take into account the eccentricities, if any, between the central line of the shell and the edge beam. Analysis should also take into account the type of edge beam, interior or exterior, as well as supporting arrangement of the edge beam.

12.1.1 Thickness

A width of two to three times the thickness of the shell subject to a minimum of 150 mm is usually necessary for the edge beams.

12.1.2 Reinforcements

Edge beams carry most of the longitudinal tensile forces due to n_x in the shell, and hence, main reinforcements have to be provided for carrying these forces. It may be necessary to provide many layers of reinforcement in the edge beam. Design of reinforcements should ensure that the stresses in the farthestmost layer does not exceed the permissible stresses. Edge beams should also be designed for carrying its self-weight, live load on the part of the shell, wind load and horizontal forces due to earthquakes.

13 DESIGN OF REINFORCEMENT

13.1 Shells

The ideal arrangement would be to lay the reinforcement in the shell to follow the isostatics, that is, directions of the principal stresses. However, for practical purposes, one of the following methods may be used:

One is the diagonal grid at 45° to the axis of the shell, and the other the rectangular grid in which the reinforcing bars run parallel to the edges of the shell. The rectangular grid needs additional reinforcement at 45° near the supports to take up the tension due to shear.

13.1.1 Placing of Reinforcement

In regions of high tension, membrane reinforcement shall, if practical, be placed in the general directions of the principal tensile membrane forces. Where this is not practical, it shall be permitted to place membrane reinforcement in two or more component directions.

Reinforcement required to resist shell bending moments shall be proportioned with due regard to the simultaneous action of membrane axial forces at the same location. Where shell reinforcement is required in only one face to resist bending moments, equal amounts shall be placed near both surfaces of the shell even though a reversal of bending moments is not indicated by the analysis.

13.1.2 In the design of the rectangular grid for cylindrical shells, the reinforcement shall usually be

divided into the following three groups:

- a) Longitudinal reinforcement to take up the longitudinal stress n_x or m_x as the case may be;
- b) Shear reinforcement to take up the principal tension caused by shear n_{xy} ; and
- c) Transverse reinforcement to resist n_y and m_y .

13.1.3 Longitudinal reinforcement shall be provided at the junction of the shell and the traverse to resist the longitudinal moment m_x . Where m_x is ignored in the analysis, nominal reinforcement shall be provided.

In case of double curvature shell like dome, reinforcements are provided in hoop and meridional/longitudinal direction. However, near crown region (depending on size of the dome), reinforcements should be provided in two orthogonal cartesian coordinate system (square mesh).

13.1.4 To ensure monolithic connection between the shell and the edge members, the shell reinforcement shall be adequately anchored into the edge members and traverses or *vice versa* by providing suitable bond bars from the edge members and traverses to lap with the shell reinforcement.

Shell reinforcement at the junction of the shell and supporting members or edge members shall be anchored in or extended through such members at least $1.2 l_d$ (development length), but not less than 450 mm.

13.2 Folded Plates

13.2.1 Transverse Reinforcement

Transverse reinforcement shall follow the cross section of the folded plate and shall be designed to resist the transverse moment.

13.2.2 Longitudinal Reinforcement

Longitudinal reinforcement, in general, may be provided to take up the longitudinal tensile stresses, in individual slabs.

13.2.3 Diagonal reinforcement for shear may be provided.

13.2.4 The section of the plate at its junction with the traverse shall be checked for shear stress caused by edge shear forces.

13.2.5 Reinforcement bars shall preferably be placed, as closely as possible, so that the steel is well distributed in the body of the plate. Nominal reinforcement consisting of minimum 8 mm diameter bars may be provided in the compression zones at about 200 mm centre-to-centre.

13.3 General

The minimum reinforcement shall conform to the requirements of IS 456. Minimum membrane reinforcement corresponding to slab shrinkage and temperature reinforcement should be provided in at least two approximately orthogonal directions even if the calculated membrane forces are compressive in one or more directions.

The area of shell tension reinforcement shall be limited, so that the reinforcement will yield before either crushing of concrete in compression or shell buckling can take place.

13.3.1 Diameters of Reinforcement Bars

The following diameters of bars may be provided in the body of the shell. Larger diameters may be provided in the thickened portions, transverse and beams:

- a) Minimum diameter — 10 mm; and
- b) Maximum diameter — one fourth of shell thickness or 16 mm whichever is smaller.

13.3.2 Spacing of Reinforcement

The maximum spacing of reinforcement in any direction in the body of the shell shall be limited to five times the thickness of the shell and the area of unreinforced panel shall in no case exceed 15 times the square of thickness.

NOTE — These limitations do not apply to edge members which are governed by IS 456.

ANNEX A

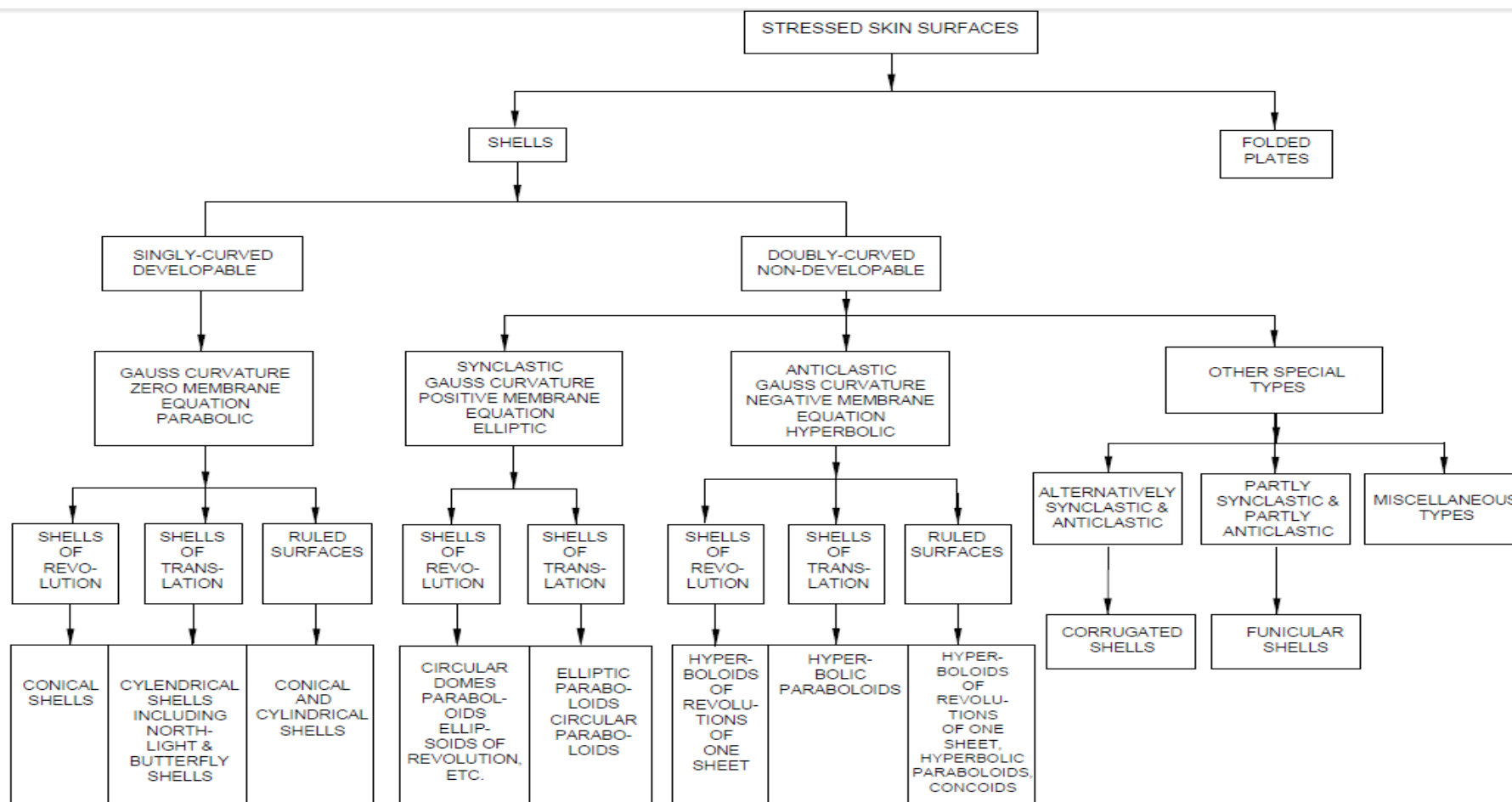
(Clause 2)

LIST OF REFERRED STANDARDS

<i>IS No.</i>	<i>Title</i>	<i>IS No.</i>	<i>Title</i>
IS 432	Mild steel and medium tensile steel bars and hard-drawn steel wire for concrete reinforcement:	(Part 2) : 1987	Live loads (<i>second revision</i>)
(Part 1) : 1982	Mild steel and medium tensile steel bars	(Part 3) : 2015	Wind loads (<i>third revision</i>)
(Part 2) : 1982	Hard-drawn steel wire	(Part 4) : 2021	Snow loads (<i>third revision</i>)
		(Part 5) : 1987	Special loads and combinations (<i>second revision</i>)
IS 456 : 2000	Plain and reinforced concrete — Code of practice (<i>fourth revision</i>)	IS 1786 : 2008	High strength deformed steel bars and wires for concrete reinforcement — Specification
IS 1343 : 2012	Prestressed Concrete — Code of practice (<i>second revision</i>)	IS 1893 (Part 1) : 2016	Criteria for earthquake resistant design of structures: Part 1 General provisions and buildings (<i>sixth revision</i>)
IS 1566 : 1982	Hard-drawn steel wire fabric for concrete reinforcement	IS 2204 : 1962	Code of practice for construction of reinforced concrete shell roof
IS 875	Code of practice for design loads (other than earthquake) for buildings and structures:	IS 6332 : 1984	Code of practice for construction of floor and roofs using precast doubly-curved shell units
(Part 1) : 1987	Dead loads (<i>second revision</i>)		

ANNEX B
(Clause 5.2)

DETAILED CLASSIFICATION OF STRESSED SKIN SURFACES



ANNEX C

(Clauses 9.2.1.3, 9.2.2.1, 9.2.2.2 and 9.2.3.1)

TABLES AND THE METHODS OF ANALYSIS OF CIRCULAR CYLINDRICAL SHELLS AND FOLDED PLATES

C-1 TABLES FOR THE ANALYSIS OF CIRCULAR CYLINDRICAL SHELLS

C-1.1 Tables given in the ASCE Manual of Engineering Practice No. 31 entitled 'Design of Cylindrical Concrete Shell Roofs', are based on a rigorous analytical method and are accurate enough for design of shells of all proportions. However, as values corresponding only to two ratios of R/d are given, interpolation involved for ratios does not give accurate results. A subsequent publication by the Portland Cement Association, Chicago, entitled 'Coefficients for Design of Cylindrical Concrete Shell Roofs', gives the coefficients for 4 values of R/d making interpolation easier.

C-1.2 Tables given in 'Circular Cylindrical Shells' by Rudiger and Urban (Published by E.G. Teubner Verlagsgesellschaft, Leipzig, 1955) are based on the

Donnel-Karman-Jenkins Method. These are particularly accurate for shells with $L/R < \pi$.

C-2 BEAM METHOD

C-2.1 This method, due to Lundgren, consists of two parts. In the first, known as the 'beam calculation', the shell is treated as a beam of curved cross section spanning between the traverses and the longitudinal stress T_x and the shear stress S are determined. In the second, known as the 'arch calculation', a unit length of the shell is treated as an arch subject to the action of applied loads and specific shear which is defined as the rate of change of shear or $\partial S / \partial x$. This calculation yields the transverse stress T and the transverse moment M_y . For a detailed treatment, reference may be made to 'Cylindrical Shells' Vol I, by H. Lundgren published by the Danish Technical Press, the Institution of Danish Civil Engineers, Copenhagen, 1949.

ANNEX D

(Clauses 9.3.1, 9.3.3.1 and 9.3.4)

GOVERNING EQUATIONS FOR ANALYSIS OF DOUBLY-CURVED SHELLS

D-1 MEMBRANE ANALYSIS

D-1.1 Real and Projected Forces and Stress Resultant

For membrane analysis of doubly-curved shells, it is usual to deal with projected stress resultants n_{xp} , n_{yp} and n_{xyp} instead of real stress resultants n_x , n_y and n_{xy} . The real and projected stress resultants are related as follows:

$$n_x = n_{xp} \sqrt{\frac{1+p^2}{1+q^2}}$$

$$n_y = n_{yp} \sqrt{\frac{1+q^2}{1+p^2}} \quad \text{and}$$

$$n_{xy} = n_{xyp}$$

Similarly, the real forces W_x , W_y and W_z on shell per unit area on its surface with x , y and z -directions are replaced by the fictitious forces X , Y and Z and the relationships between them are as follows:

$$X = W_x \sqrt{1+p^2+q^2}$$

$$Y = W_y \sqrt{1+p^2+q^2} \quad \text{and}$$

$$Z = W_z \sqrt{1+p^2+q^2}$$

D-1.2 Equilibrium Equations

The three equations of equilibrium can now be written using the projected stress resultants and fictitious forces as follows:

$$\frac{\partial n_{xp}}{\partial x} + \frac{\partial n_{xyp}}{\partial y} + X = 0 \quad \dots(1)$$

$$\frac{\partial n_{yp}}{\partial y} + \frac{\partial n_{xyp}}{\partial x} + Y = 0 \quad \text{and} \quad \dots(2)$$

$$rn_{xp} + 2sn_{xyp} + tn_{yp} = p_x + q_y - Z \quad \dots(3)$$

D-1.3 Analysis Using Stress Function

Analysis of the equations of equilibrium is simplified by using a stress function ϕ which reduces the three equations to one. The stress function ϕ is related to the projected stress

resultants as follows:

$$n_{xp} = \frac{\partial^2 \phi}{\partial y^2} - \int X dx$$

$$n_{yp} = \frac{\partial^2 \phi}{\partial x^2} - \int Y dy$$

and

$$n_{xyp} = \frac{-d^2 \phi}{\partial x \partial y} \quad \dots(4)$$

On introducing the stress function, the third equilibrium equation reduces to the following:

$$\begin{aligned} & r \left[\frac{\partial^2 \phi}{\partial y^2} \right] - 2s \left[\frac{\partial^2 \phi}{\partial x \partial y} \right] + t \left[\frac{\partial^2 \phi}{\partial x^2} \right] \\ & pX + qY - Z + r \int X dx + t \int Y dy \end{aligned} \quad \dots(5)$$

The homogeneous part of the partial differential equation given above will be of the elliptic, parabola or hyperbolic type depending upon whether $s^2 - r.t \lessgtr 0$, which is also the test for classifying shells as synclastic, developable or anticlastic.

D-2 BENDING ANALYSIS

D-2.1 Bending analysis is necessary for shallow doubly-curved shells. For shallow shells under vertical loading, this would involve the solution, of two partial simultaneous equations given below:

a) for shells of variable curvature:

$$\left. \begin{aligned} & \frac{1}{E_{cd}} \nabla^4 F + \left(\frac{\partial}{\partial x} t \frac{\partial}{\partial x} - 2s \frac{\partial^2}{\partial x \partial y} + \frac{\partial r}{\partial y} \frac{\partial}{\partial y} \right) w = 0 \\ & \text{and} \\ & D \nabla^4 w - \left(\frac{\partial}{\partial x} t \frac{\partial}{\partial x} - 2s \frac{\partial^2}{\partial x \partial y} + \frac{\partial r}{\partial y} \frac{\partial}{\partial y} \right) F - Z = 0 \end{aligned} \right\} \dots (6)$$

b) For shells of constant curvature, that is, for shells for which r , s and t are constant, above equations simplify to:

$$\left. \begin{aligned} & \frac{1}{E_{cd}} \nabla^4 F + \left(t \frac{\partial^2 w}{\partial x^2} - 2s \frac{\partial^2 w}{\partial x \partial y} + r \frac{\partial^2 w}{\partial y^2} \right) = 0 \\ & \text{and} \\ & D \nabla^4 w - \left(t \frac{\partial^2 F}{\partial x^2} - 2s \frac{\partial^2 F}{\partial x \partial y} + r \frac{\partial^2 F}{\partial y^2} \right) - Z = 0 \end{aligned} \right\} \dots (7)$$

where

F = Stress function which gives in-plane stresses, when bending is also considered;

w = Deflection along z-axis;

D = Flexural rigidity = $E_c d^3 / \{12(1-\nu^2)\}$;

Z = Vertical load per unit area of shell surface, assumed positive in the positive direction of Z-axis;

ν = Poisson ratio; and

r , s and t are curvature as defined in 4.1.

The equations given above are based on the co-ordinate system in Fig. 9.

D-2.2 Bending Stress Resultants

From the values of F and w satisfying equations (6) or (7), the stress resultants may be obtained from the following relations:

$$\begin{aligned} T_x &= \frac{\partial^2 F}{\partial y^2} \\ T_y &= \frac{\partial^2 F}{\partial x^2} \\ S &= -\frac{\partial^2 F}{\partial x \partial y} \\ m_x &= D \left[\frac{\partial^2 w}{\partial x^2} + \nu \frac{\partial^2 w}{\partial y^2} \right] \\ m_y &= D \left[\frac{\partial^2 w}{\partial y^2} + \nu \frac{\partial^2 w}{\partial x^2} \right] \\ m_{xy} &= -D(1-\nu) \frac{\partial^2 w}{\partial x \partial y} \end{aligned}$$

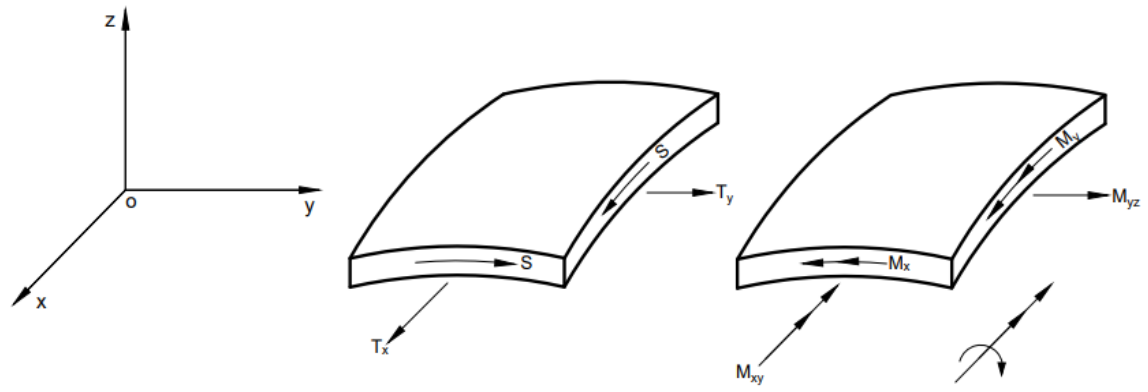


FIG. 9 SIGN CONVENTION FOR STRESSES AND MOMENTS IN A SHELL ELEMENT

ANNEX E

(Clause 9.1.2)

GENERAL SOLUTION FOR TWO-WAY SLABS, WALLS AND SHELL ELEMENTS

E-1 GENERAL

This Section deals with plate and shell type elements and sub-elements where out of plane loading effects as well as in plane loading effects are present. The permissible simplifications for separating in plane and out of plane effects and designing for the same are given. A method of providing reinforcement for in plane effects is indicated. For use of three dimensional elements as bridge elements (For example, shell elements as in case of fish-belly superstructure) apart from designing for sectional strength, overall and local buckling checks may be required. Specialist literature may be referred for the same.

E-2 ONE-WAY AND TWO-WAY SLABS AND WALLS

For predominantly transverse loads acting perpendicular to the plane of the slab, where primary overall in plane tensile or compressive stress fields are absent, the slabs / walls can be designed using conventional plate bending analysis and providing ultimate strength in bending and shear in one and/or two orthogonal directions as required by such analysis. Bridge deck slabs, retaining walls resisting lateral earth pressures etc., are the typical examples.

Any rational method of analysis which does not permit more than 15 percent redistribution of peak bending moments over supports can be used. The ultimate strength methods based on local yielding (for example, yield line method) are not permitted. However, for calculating resistance to accidental impact loads (for example, vehicular or barge impacts), use of such methods is permitted for which specialist literature may be referred.

E-3 SUB-ELEMENTS OF BOX STRUCTURES

- a) Where box type structure resists the overall longitudinal bending, shear and torsional effects, significant in-plane longitudinal stresses are present in its sub-elements, which coexist with the local out of plane loading, for example, effects due to self weight, live loads, intermediate prestressing anchorages etc.
- b) Such elements can be designed primarily for in-plane effects arising from overall bending, shear and torsion in longitudinal direction. In transverse direction they are designed for out of plane loading effects.

- c) If effects of out-of-plane loading are present in the same direction as of the overall in-plane tensions/compressions, these elements should be locally checked for combined effect of in-plane forces, and out of plane bending moments and shears.

E-4 GENERAL SOLUTION FOR TWO-WAY SLABS, WALLS AND SHELL ELEMENTS

For plate type elements having complex geometry and edge conditions and having in-plane as well as out-of-plane (transverse) effects due to loading, the resultant stress fields may be obtained by use of suitable finite elements in the FEM analysis. The analysis yields resultant stresses which represent combined effects of in-plane membrane stresses and local bending effects. These stress resultants can be directly used to design tensile reinforcement and for verifying safety in compression. A general method of design using sandwich model given in Annex F may be referred. Alternatively, the stresses can be converted to equivalent axial forces and bending effects in orthogonal directions and used for designing the sections following conventional methods given in E-4.1, E-4.2 and E-4.3.

E-4.1 Simplified Design for Tensile Reinforcement for Orthogonal In-Plane Effects

The following simplified method may be used for proportioning of tensile reinforcement based on the in-plane stresses $\sigma_{Edx} > \sigma_{Edy}$ and τ_{Edxy}

- a) Compressive stresses should be taken as positive, with $\sigma_{Edx} > \sigma_{Edy}$, and the direction of reinforcement should coincide with the x and y axes.
- b) The tensile strengths f_{tdx} and f_{tdy} provided by reinforcement should be determined from:

$$f_{tdx} = \rho_x \cdot f_{yd} \text{ and } f_{tdy} = \rho_y \cdot f_{yd}$$

Where ρ_x and ρ_y are the geometric reinforcement ratios, along the x and y axes respectively.

- c) In locations where σ_{Edx} and σ_{Edy} are both compressive and $\sigma_{Edx} \cdot \sigma_{Edy} > \tau_{Edxy}^2$, design reinforcement is not required. However, the maximum compressive stress should not exceed f_{cd} .

- d) In locations where σ_{Edy} is tensile, or $\sigma_{Edx} \cdot \sigma_{Edy} \leq \tau_{Edxy}^2$ reinforcement is required.
- e) The optimum reinforcement, indicated by superscript (') and placed in directions of concerned principal stresses are determined by the following equations:

$$\text{For } \sigma_{Edx} \leq |\tau_{Edxy}|$$

$$f'_{tdx} = |\tau_{Edxy}| - \sigma_{Edx} \quad \dots(9)$$

$$f'_{tdy} = |\tau_{Edxy}| - \sigma_{Edy} \quad \dots(10)$$

$$\sigma_{cd} \leq 2|\tau_{Edy}| \quad \dots(11)$$

$$\text{For } \sigma_{Edx} > |\tau_{Edxy}|$$

$$f'_{tdx} = 0 \quad \dots(12)$$

$$f'_{tdx} = \frac{\tau_{Edxy}^2}{\sigma_{Edx}} - \sigma_{Edy} \quad \dots(13)$$

$$\sigma_{cd} = \sigma_{Edx} \left(1 + \left(\frac{\tau_{Edxy}}{\sigma_{Edx}} \right)^2 \right) \quad \dots(14)$$

The concrete stress σ_{cd} , should not exceed $\nu \cdot f_{cd}$. The minimum reinforcement is obtained if the direction of reinforcement is identical to the direction of principal stresses, where value of ν is obtained from the equation is given below:

$$\nu = 0.6 \left[1 - \frac{f_{ck}}{310} \right]$$

where, f_{ck} is in MPa.

Alternatively, for the general case the necessary reinforcement and the concrete stress can be determined by:

$$f_{tdx} = |\tau_{Edxy}| \cot \theta - \sigma_{Edx} \quad \dots(15)$$

$$f_{tdy} = |\tau_{Edxy}| / \cot \theta - \sigma_{Edy} \quad \dots(16)$$

$$\sigma_{cd} = |\tau_{Edxy}| \left(\cot \theta + \frac{1}{\cot \theta} \right) \quad \dots(17)$$

where (θ) is the angle of the concrete compressive stress to the x-axis. The value of $\cot(\theta)$ shall be chosen to avoid compressive value of f_{td} .

NOTES

1 In order to avoid unacceptable cracks for the serviceability state, and to ensure the required deformation capacity for the ultimate limit state, the reinforcement derived from equations (15) and (16) for each direction should not be more than thrice and not less than half the reinforcement determined by equations (9) and (10) or, alternatively, from equations (12) and (13). These limitations are expressed by:

$$\frac{1}{2} f'_{tdx} \leq f_{tdx} \leq 2 f'_{tdy} \text{ and } \frac{1}{2} f'_{tdy} \leq f_{tdy} \leq 2 f'_{tdx}$$

2 The reinforcement should be fully anchored at all free edges.

E-4.2 Simplified Design for Bending in Orthogonal Direction

The slab (wall) subjected to orthogonal bending effects is substituted by two half plates usually representing the compression and tensile zones of the element.

The stresses in compressive portions are checked by verifying that the resultant principle compressive stress is within the allowable limits.

The half plate carrying tensile stresses in orthogonal directions is converted to orthogonal mesh of steel following method described in E-4.1.

The out of plane shear forces are similarly converted to principle bending directions and the maximum shear.

E-4.3 Simplified Design of Combined In-Plane Forces and Out of Plane Bending and Shears

The plate is substituted by a sandwiched plate of the same thickness consisting of three layers of 1/3rd thickness. The central 1/3rd thickness is designed to carry in-plane forces using methods of E-4.1.

The outer two layers are designed to resist tensile and compressive fields with appropriate lever-arm to develop resistance to bending and shear in two directions as per E-4.2.

ANNEX F

(Clauses 9.1.2 and E-4)

GUIDE TO FINITE ELEMENT MODELING OF REINFORCED CONCRETE STRUCTURES

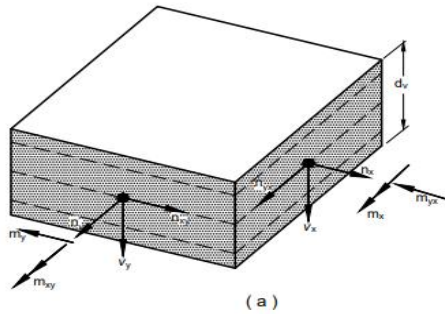
F-1 SANDWICH MODEL FOR THE DIMENSIONING OF SHELL ELEMENTS

The eight stress resultants acting on a shell element, Fig. 10A, can be resisted by a sandwich model. In this model the bending and twisting moments m_x , m_y , m_{xy} as well as the in-plane forces n_x , n_y , n_{xy} are attributed to the sandwich covers, while the sandwich core resists the transverse shear forces v_x , v_y (illustrated in Fig. 10B). Thus, the sandwich model represents a lower bound solution with its basis in plasticity theory.

The sandwich core transfers the principal transverse shear force $v_0 = \sqrt{v_x^2 + v_y^2}$ in the direction $\varphi_0 = \tan^{-1}(v_x/v_y)$.

If the transverse shear forces are small, that is, nominal shear stresses v_0/d_v are below the nominal shear cracking resistance, $\tau_{c,red}$, the core will remain uncracked and the forces in the bottom and top sandwich covers follow directly as:

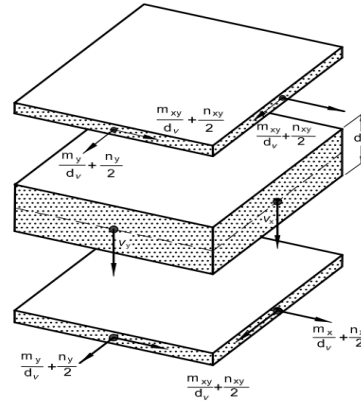
$$\left. \begin{aligned} n_{x \text{ inf,sup}} &= \pm \frac{m_x}{d_v} + \frac{n_x}{2} \\ &\text{(uncracked core, } v_0/d_v \geq \tau_{c,red}) \\ n_{y \text{ inf,sup}} &= \pm \frac{m_y}{d_v} + \frac{n_y}{2} \\ n_{xy \text{ inf,sup}} &= \pm \frac{m_{xy}}{d_v} + \frac{n_{xy}}{2} \end{aligned} \right\} \dots (18)$$



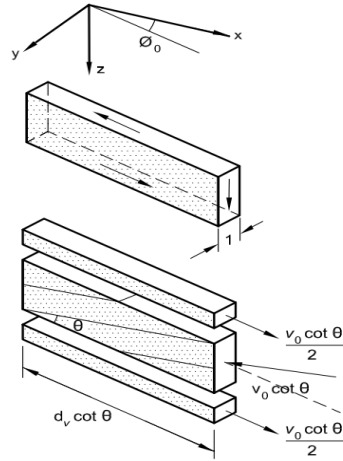
10A SHELL ELEMENT

If the principal transverse shear force $v_0 = \sqrt{v_x^2 + v_y^2}$ is high enough to produce cracking of the sandwich core, the latter can be treated like the web of a girder of flanged cross-section running in direction q_0 (see Fig. 10). The corresponding tensile forces in the element plane must be resisted by the sandwich covers in addition to the forces given by equation (18), resulting in the following total forces in the sandwich covers.

$$\left. \begin{aligned} n_{x \text{ inf,sup}} &= \pm \frac{m_x}{d_v} + \frac{n_x}{2} + \frac{v_x^2}{2v_0 \tan \theta} \\ n_{y \text{ inf,sup}} &= \pm \frac{m_y}{d_v} + \frac{n_y}{2} + \frac{v_y^2}{2v_0 \tan \theta} \\ &\text{(cracked core, } v_0/d_v \geq \tau_{c,red}) \\ n_{xy \text{ inf,sup}} &= \pm \frac{m_{xy}}{d_v} + \frac{n_{xy}}{2} + \frac{v_x v_y}{2v_0 \tan \theta} \end{aligned} \right\} \dots (19)$$



10B LAYER FORCES



10C TRANSFER OF TRANSVERSE SHEAR FORCE IN UNCRACKED AND CRACKED CORE

FIG. 10 SANDWICH MODEL

According to equations (18) and (19), a state of plane stress is obtained in the sandwich covers, and they can be dimensioned as membrane elements. For the case of failure governed by yielding of the reinforcement with the concrete remaining elastic in both sandwich covers, the force per unit width in the reinforcement in the x and y directions are determined with:

$$\left. \begin{aligned} a_{sx}f_y &\geq \frac{m_x}{d_v} + \frac{n_x}{2} + \frac{v_x^2}{2v_0 \tan \theta} + k \left| \frac{m_{xy}}{d_v} + \frac{n_{xy}}{2} + \frac{v_x v_y}{2v_0 \tan \theta} \right| \\ a_{sy}f_y &\geq \frac{m_y}{d_v} + \frac{n_y}{2} + \frac{v_y^2}{2v_0 \tan \theta} + k^{-1} \left| \frac{m_{xy}}{d_v} + \frac{n_{xy}}{2} + \frac{v_x v_y}{2v_0 \tan \theta} \right| \\ a'_{sx}f_y &\geq -\frac{m_x}{d_v} + \frac{n_x}{2} + \frac{v_x^2}{2v_0 \tan \theta} + k' \left| -\frac{m_{xy}}{d_v} + \frac{n_{xy}}{2} + \frac{v_x v_y}{2v_0 \tan \theta} \right| \\ a'_{sy}f_y &\geq -\frac{m_y}{d_v} + \frac{n_y}{2} + \frac{v_y^2}{2v_0 \tan \theta} + k'^{-1} \left| -\frac{m_{xy}}{d_v} + \frac{n_{xy}}{2} + \frac{v_x v_y}{2v_0 \tan \theta} \right| \end{aligned} \right\} \dots(20)$$

where, a_s and a'_s are the bottom and top reinforcement areas per unit width of the slab, respectively.

For an uncracked core, that is $v_0/d_v < t_{c,red}$, the terms in equation (20) containing v_x or v_y can be omitted and no transverse shear reinforcement is required. That is $r_z = 0$. If the core is cracked, $v_0/d_v > t_{c,red}$, transverse reinforcement of

$$\rho_z = \frac{v_0 \tan \theta}{d_v f_y} \dots(21)$$

is required for the core to carry transverse shear force. In equations (18) to (21), d_v is the effective depth of the membrane forces in the sandwich covers, k and k' are, arbitrary positive factors and theta (θ) is the inclination of the diagonal compression field in the sandwich core. Different values of k and k' can be adopted for each element of a shell or slab, avoiding abrupt changes or providing sufficient

development length in order to anchor the differential forces in the reinforcement while in choosing the inclination angle, theta (θ). In this regard similar considerations apply as in the design of membranes.

For simplicity in design practice using, $k = k' = 1$, and theta (θ) = 45° often provides for a good solution.

Equation (20) is valid only if the concrete in the sandwich covers does not crush. In order to satisfy this condition, the thickness of the sandwich covers, and consequently the effective depth d_v , should be chosen such that

$$\left. \begin{aligned} f_c t_{inf} &\geq a_{sx}f_y + a_{sy}f_y - (n_{x,inf} + n_{y,inf}) \\ f_c t_{sup} &\geq a'_{sx}f_y + a'_{sy}f_y - (n_{x,sup} + n_{y,sup}) \end{aligned} \right\} \dots(22)$$

Where, t_{inf} and t_{sup} = thickness of bottom and top sandwich cover and f_c = effective concrete compressive strength, respectively. Substituting equations (19) and (20) in equation (21), this can be written as:

$$\left. \begin{aligned} f_c t_{inf} &\geq (k + k^{-1}) \left| \frac{m_{xy}}{d_v} + \frac{n_{xy}}{2} + \frac{v_x v_y}{2v_0 \tan \theta} \right| \\ f_c t_{sup} &\geq (k + k^{-1}) \left| -\frac{m_{xy}}{d_v} + \frac{n_{xy}}{2} + \frac{v_x v_y}{2v_0 \tan \theta} \right| \end{aligned} \right\} \dots(23)$$

As a first and often sufficient approximation, d_v can be chosen such that the middle planes of each sandwich cover coincides with the centre between its two reinforcement layers.

The threshold value $\tau_{c,red}$ for considering cracked or uncracked behaviour of the sandwich core should be carefully selected. Basically it could be taken as the nominal shear strength of the slab or shell without

transverse reinforcement, which, due to size effect, varies with the element thickness; for example, $\tau_{c,red} = f_{ctd}$ for membrane shear according to [CEB-FIP (1990)]. Since the provision of transverse reinforcement is quite time-consuming, it is advisable to choose the element thickness such that, apart from zones where concentrated forces are introduced, no transverse reinforcement is required.

In cases with high axial compression n_x , n_y or high in-plane shear forces n_{xy} , it is possible to attribute part of these forces to the sandwich core and then, strictly speaking, the concrete compression in the sandwich core must be checked taking into account the combined action of (n_x, n_y, n_{xy}) and v_0 . The case of high axial compression is likely to occur if prestressing is treated as forces acting on the structure. However, this is beyond the scope of this report.

F-2 DIMENSIONING OF SLAB AND SHELL ELEMENTS IN DESIGN PRACTICE

The design equations derived from the sandwich model [equations (20) and (21)] are simple and transparent, allowing for a straightforward dimensioning of in-plane as well as transverse reinforcement in the general case of a shell element subjected to eight independent stress resultants. In the special case of slabs and neglecting transverse shear forces, equation (20) is equivalent to the normal moment yield criterion [equation (13)] if exact values are taken for d_v .

The incorporation of equations (20) and (21) into a finite element program for shells does not present any more difficulties than the implementation of the normal moment yield criterion for the dimensioning of reinforcement, a post-processing option which is already offered by a number of commercial programs today (often called ‘Wood-Armer’ moments).

Usually, no explicit check of the assumed failure mode is made in slabs, assuming implicitly that the reinforcement ratios obtained are small enough to avoid concrete crushing. While this condition is usually satisfied in slabs, a check of the assumed failure mode should always be carried out for shells, particularly for elements subjected to significant axial compression or cases with high reinforcement ratios. Thus, it is advisable to implement the corresponding check of concrete resistance according to equation (22) together with equations (20) and (21).

Transverse shear forces obtained from a finite element calculation will generally be less accurate than bending and twisting moments, which can be rewritten as, $v_x = m_{x,x} + m_{xy,y}$ and $v_y = m_{y,y} + m_{yx,x}$, transverse shear forces are derivatives of the bending and twisting moments. Thus, in order to get reasonable results, a finer element mesh is required when using equations (20) and (21) in their general form than in cases where transverse shear forces can be neglected a priori.

ANNEX G (Foreword)

RESPONSE TRANSFORMATION OF RESPONSE FROM NUMERICAL METHODS

G-1 GENERAL

This Annex deals with the suggested methodology for estimating design reinforcement, based on finite element methods, or other numerical techniques.

With the advent use of numerical methods, such as, finite element analysis, finite difference analysis, etc, responses obtained from these methods should be properly accounted, while evaluating final design of plates and shells. In-plane forces obtained along the principal directions of finite elements may be transformed to obtain principal directions, to have severe effect due to applied loading. Principal directions of the forces may not coincide with the orthogonal or non-orthogonal principal directions of finite elements and directions of reinforcement to be provided within the shell.

Wood's criterion is one of the acceptable methods to transform elemental responses to principal directions. Final reinforcement direction may not coincide with the principal directions. Thus, reinforcement may be estimated in principal directions, and then transformed to orthogonal directions (for folded plates) or meridional and circumferential directions (for single or double curved shell).

Out of various available methods for estimating reinforcement in plates and shells, two methods are presented in G-2 and G-3.

G-2 ANALYTICAL METHOD

G-2.1 The method of calculating plate and shell reinforcement is based on the conception presented in the A. Capra and J. F. Maury's article titled "*Calcul automatique du ferrailage optimal des plaques et coques en beton arme*", *Annales de l'Institut Technique du Batiment et des Travaux Publics*, No. 367, December 1978.

G-2.2 Calculation Procedure

If the reinforcement values A_x and A_y (corresponding to two perpendicular directions 'x' and 'y') are given, an equivalent reinforcement in any other direction (n) according to the following formula:

$$A_n = A_x * \cos^2(\alpha) + A_y * \sin^2(\alpha)$$

where ' α ' is an angle included between the direction 'x' and the direction 'n'.

The values of sectional forces (moments and membrane forces) M_n , N_n may be obtained from the following transformational formulas,

$$M_n = M_x * \cos^2(\alpha) + M_y * \sin^2(\alpha) - M_{xy} * \sin^2(2\alpha)$$

$$N_n = N_x * \cos^2(\alpha) + N_y * \sin^2(\alpha) - N_{xy} * \sin^2(2\alpha)$$

Thus, the below-presented inequality formulates the condition of correct reinforcement. The reinforcement that is able to carry the internal forces in an arbitrary section,

$$A_x * \cos^2(\alpha) + A_y * \sin^2(\alpha) = A_n \geq \emptyset (M_n, N_n)$$

where, $\emptyset (M_n, N_n)$ refers to the value of reinforcement required to carry the forces calculated for the direction 'n' - M_n, N_n .

Inequality

$$A_x * \cos^2(\alpha) + A_y * \sin^2(\alpha) \geq \emptyset (M_n, N_n)$$

This determines on the plane (A_x, A_y) the area of 'admissible' values of reinforcement A_x, A_y (half-plane). If such area is determined for a sufficiently 'dense' set of directions n (control is performed every 10), one obtains the area of admissible values A_x, A_y .

The adopted reinforcement is the minimal reinforcement which yields the minimal sum of surfaces $A_x + A_y$.

If a structure type or selection of calculation options causes reduction of internal forces, the reinforcement is calculated based on the following:

- a) M_n moments. Plate structure or simple bending option in a shell structure;
- b) N_n membrane forces. Plane stress structure or compression/tension option in a shell structure; and
- c) Complete set of M_n, N_n forces. Bending + compression/tension option in a shell structure.

Note that in calculations of unidirectional reinforcement, the analytical method is limited to calculating the reinforcement only for the main reinforcement direction, without dividing it into n directions. A plate is designed only for the M_{xx} and N_{xx} set of forces.

G-3 WOOD AND ARMER METHOD

G-3.1 Wood and Armer method (*the European code supplement [ENV 1992-1-1 EC2 Design of Concrete Structures - Annex 2, point A.2.8 Reinforcement in Slabs]*) can be used for transformation of responses. The conception of determining equivalent moment is authored by Wood and Armer. Details concerning the method can be found, for instance, in R.H. Wood, "The reinforcement of slabs in accordance with a pre-determined field of moments", Concrete, February 1968, August 1968 (correspondence)].

G-3.2 Calculation Procedure

When calculating reinforcement of a plate structure or switching on the option of panel design for simple bending in a shell structure, design moments are calculated according to the method by Wood and Armer.

- a) For a selected directions x and y , two types of design moments M^* are calculated: the lower ones (positive, causing mainly tension in the bottom parts) and the upper ones (negative, causing tension in the upper parts). The general procedure takes the following form:

1) *Determination of the 'lower' moments M_{xd}^* , M_{yd}^**

$$M_{xd}^* = M_x + |M_{xy}|$$

$$M_{yd}^* = M_y + |M_{xy}|$$

However, if $M_x < -|M_{xy}|$ (that is, the calculated $M_{xd}^* < 0$)

$$M_{xd}^* = 0$$

$$M_{yd}^* = M_y + |M_{xy} * M_{xy} / M_x|$$

Similarly, when $M_y < -|M_{xy}|$ (that is, the calculated $M_{yd}^* < 0$) $\rightarrow (*)$

$$M_{xd}^* = M_x + |M_{xy} * M_{xy} / M_y| \rightarrow (*)$$

$$M_{yd}^* = 0 \rightarrow (*)$$

If any of these obtained moments M_{xd}^* , M_{yd}^* is smaller than zero, you should assume zero (the design moments for tension in the upper layers are determined further on in the text).

2) *Determination of the 'upper' moments M_{xg}^* , M_{yg}^**

$$M_{xg}^* = M_x - |M_{xy}|$$

$$M_{yg}^* = M_y - |M_{xy}|$$

If $M_x > |M_{xy}|$ (that is, the calculated $M_{xg}^* > 0$) $\rightarrow (*)$

$$M_{xg}^* = 0 \rightarrow (*)$$

$$M_{yg}^* = M_y - |M_{xy} * M_{xy} / M_x| \rightarrow (*)$$

Similarly, when $M_y > |M_{xy}|$ (that is, the calculated $M_{yg}^* > 0$)

$$M_{xg}^* = M_x - |M_{xy} * M_{xy} / M_y|$$

$$M_{yg}^* = 0$$

If any of these obtained moments M_{xg}^* , M_{yg}^* is greater than zero, assume zero (such moments would design the lower reinforcements, which is already guaranteed by the formerly calculated 'lower' moments M_{xd}^* , M_{yd}^*).

Analogously, design forces are calculated from the formulas given below for a plane stress structure or for the activated option of panel design for compression/ tension in a shell structure.

- b) For the selected directions x and y , two types of design forces N^* are calculated: the tensile (positive, causing main tension in a section) and the compressive (negative, causing section compression). The general procedure takes the following form:

1) *Calculation of 'tensile' forces N_{xr}^* , N_{yr}^**

$$N_{xr}^* = N_x + |N_{xy}|$$

$$N_{yr}^* = N_y + |N_{xy}|$$

However, if $N_x < -|N_{xy}|$ (that is, calculated $N_{xr}^* < 0$)

$$N_{xr}^* = 0$$

$$N_{yr}^* = N_y + |N_{xy} * N_{xy} / N_x|$$

Similarly, if $N_y < -|N_{xy}|$ (that is, calculated $N_{yr}^* < 0$) $\rightarrow (*)$

$$N_{xr}^* = N_x + |N_{xy} * N_{xy} / N_y| \rightarrow (*)$$

$$N_{yr}^* = 0 \rightarrow (*)$$

If any of these obtained forces N_{xr}^* , N_{yr}^* is less than zero, assume these as zero value (forces designing a section by reinforcement compression are determined further on).

b) *Calculation of 'compressive' forces N_{xs}^* , N_{ys}^**

$$N_{xs}^* = N_x - |N_{xy}|$$

$$N_{ys}^* = N_y - |N_{xy}|$$

However, if $N_x > |N_{xy}|$ (that is, calculated $N_{xs}^* > 0$) $\rightarrow (*)$

$$N_{xs}^* = 0 \rightarrow (*)$$

$$N_{ys}^* = N_y - |N_{xy}^* N_{xy} / N_x| \rightarrow (*)$$

Similarly, if $N_y > |N_{xy}|$ (that is, calculated $N_{ys}^* > 0$)

$$N_{xs}^* = N_x - |N_{xy}^* N_{xy} / N_y|$$

$$N_{ys}^* = 0$$

If any of these obtained forces N_{xs}^* , N_{ys}^* is greater than zero, assume these zero as value (such forces design a section by reinforcement tension, which is already guaranteed by the tensile forces N_{xr}^* , N_{yr}^* calculated earlier).

For complex stresses (shells with the activated option of panel design for bending + compression/tension) with bending moments (M_{xx} , M_{xy} , M_{yy}) and membrane forces (N_{xx} , N_{xy} , N_{yy}) acting simultaneously, there is no simplified algorithm devised. Since it is often the case that the modeled shells work almost as plates (with slight membrane forces acting), therefore the possibility to calculate moments M_{xd}^* , M_{yd}^* according to the method presented still remains and these design moments are superimposed with longitudinal forces N_{xx} , N_{yy} .

ANNEX H

(Foreword)

COMMITTEE COMPOSITION

Special Structures Sectional Committee, CED 38

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In the formulation of this standard due weightage has been given to international coordination among the standards and practices prevailing in different countries in addition to relating it to the practices in this field in the country. For this purpose, assistance has been derived from the following publications:

ACI 318 : 2019 Building code requirements for structural concrete and commentary (American Concrete Institute)

ACI 334.1 Concrete Shell Structures — Practice and Commentary (American Concrete Institute)

FIB Bulletin 45 — Practitioner's guide to finite element modelling of reinforced concrete structures (The International Federation for Structural Concrete)

IRC 112 : 2011 Code of practice for concrete road bridges (Indian Road Congress)

The composition of the Committee responsible for the formulation of this standard is given in Annex H.

For the purpose of deciding whether a particular requirement of this standard is complied with, the final value, observed or calculated, expressing the result of a test or analysis, shall be rounded off in accordance with IS 2 : 2022 'Rules for rounding off numerical values (*second revision*)'. The number of significant places retained in the rounded off value should be the same as that of the specified value in this standard.

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Amendments Issued Since Publication

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